Mathematical Exercises

1. Calculate the ultrasound wavelength in typical soft tissue ($c = 1540$ m/sec) for frequencies of 2, 5 and 10 MHz.

2. In an ultrasound system the ratio of the intensities of the strongest to the weakest echo is 320,000. What is the dynamic range in decibels?

3. Calculate the round path attenuation in typical soft tissue ($\alpha = 0.5$ dB/cm/MHz) for frequencies of 2, 5 and 15 MHz and (for each frequency) depths of 1, 10 and 20 cm (i.e. a total of nine different frequency-depth combinations).

4. If a 10 MHz probe has a depth of penetration of 6 cm, what would you expect the penetration depth would be for a 2.5 MHz probe?

5. What is the reflection coefficient (assuming perpendicular incidence) for an interface between two tissues with acoustic impedances of 1.3×10^6 and 1.6×10^6 Rayl?

6. Calculate the transmission angle for an incident angle of 60° at an interface between fat ($c_1 = 1450$ m/sec) and soft tissue ($c_2 = 1540$ m/sec). Does this interface have a critical angle?

7. Calculate the time delay between the transmit pulse and the reception of the echo for structures at 1, 10 and 20 cm depth.

8. Calculate the maximum allowable PRF for penetration depths of 1, 10 and 20 cm.

9. Calculate the maximum PRF possible for a transducer with a depth of penetration of 15 cm. If each image requires 140 lines of sight and there is one transmit pulse for each line of sight, calculate the maximum possible frame rate.

10. For unfocused transducers with frequencies of 2, 5 and 15 MHz and (for each frequency) apertures of 2, 10 and 20 mm (i.e. a total of nine different frequency-aperture combinations) calculate (a) the length of the near zone and (b) the diffraction limit. Comment on the practical implications of these results.

11. (a) Calculate the beamwidth at focus for a 4 MHz transducer with an aperture of 2 cm focussed at a depth of 5 cm. (b) Repeat the calculation for the case where the same transducer is focussed at a depth of 15 cm.

12. Ultrasound is used to image a tissue interface at a depth of 8 cm. Most of the tissue through which the ultrasound passes has a propagation speed of 1540 m/sec, but there is a fat layer of 2 cm thickness adjacent to the face of the transducer with a propagation speed of 1450 m/sec. At what depth will the machine display the tissue interface?

13. What is the lateral and axial resolution for a transducer with a beamwidth of 5 mm and a transmit pulse duration of 1.5 μsec?

14. Calculate the Doppler shift for a 4 MHz Doppler system, a blood velocity of 80 cm/sec and a Doppler angle of 60°.

15. A 2.5 MHz pulsed Doppler is operating with a PRF of 4 kHz. (a) What is the Nyquist Limit? (b) If $\theta = 30°$, what blood velocity would cause a Doppler shift equal to this Nyquist Limit?