Mathematical Exercises - answers

1. Calculate the ultrasound wavelength in typical soft tissue (c = 1540 m/sec) for frequencies of 2, 5 and 10 MHz.

$\lambda = c/f$ = 0.77 mm, 0.31 mm, 0.15 mm.

2. In an ultrasound system the ratio of the intensities of the strongest to the weakest echo is 320,000. What is the dynamic range in decibels?

Dynamic range = 10 log (320,000) = 55 dB.

3. Calculate the round path attenuation in typical soft tissue (α = 0.5 dB/cm/MHz) for frequencies of 2, 5 and 15 MHz and (for each frequency) depths of 1, 10 and 20 cm (i.e. a total of nine different frequency-depth combinations).

Round path attenuation = $2 \times \alpha \times d \times f$

	d = 1 cm	d = 10 cm	d = 20 cm
f = 2 MHz	2 dB	20 dB	40 dB
f = 5 MHz	5 dB	50 dB	100 dB*
f = 15 MHz	15 dB	150 dB*	300 dB*

**unacceptably high attenuation, so beyond the penetration depth*

4. If a 10 MHz probe has a depth of penetration of 6 cm, what would you expect the penetration depth would be for a 2.5 MHz probe?

$(P \times f)$ = constant

so (6 cm × 10 MHz) = (P_2 × 2.5 MHz)

and so $P_2 = (6 \times 10)/2.5 = 24$ cm

5. What is the reflection coefficient (assuming perpendicular incidence) for an interface between two tissues with acoustic impedances of 1.3×10^6 and 1.6×10^6 Rayl?

R = 0.011 = 1.1%

6. Calculate the transmission angle for an incident angle of 60° at an interface between fat (c_1 = 1450 m/sec) and soft tissue (c_2 = 1540 m/sec). Does this interface have a critical angle?

θ_t = 66.9°.

Yes, since the propagation speed is higher in the second tissue there will be a critical angle.

7. Calculate the time delay between the transmit pulse and the reception of the echo for structures at 1, 10 and 20 cm depth.

Using the approximation that the echo delay is 13 µsec for each 1 cm of depth, the delays are: 13 µsec, 130 µsec, 260 µsec.

8. Calculate the maximum allowable PRF for penetration depths of 1, 10 and 20 cm.

c/2P = 77 kHz, 7.7 kHz, 3.9 kHz.

9. Calculate the maximum PRF possible for a transducer with a depth of penetration of 15 cm. If each image requires 140 lines of sight and there is one transmit pulse for each line of sight, calculate the maximum possible frame rate.

c/2P = 5.1 kHz.

This means there are 5,100 transmit pulses each second.

Since it takes 140 transmit pulses to create each image, the number of images per second (i.e. the frame rate) is
FR = (5100/140) = 36 Hz

10. For unfocused transducers with frequencies of 2, 5 and 15 MHz and (for each frequency) apertures of 2, 10 and 20 mm (i.e. a total of nine different frequency-aperture combinations) calculate (a) the length of the near zone and (b) the diffraction limit. Comment on the practical implications of these results.

(a) Near Zone Length:

	A = 1 mm	A = 10 mm	A = 20 mm
f = 2 MHz	0.13 cm*	3.2 cm*	13 cm
f = 5 MHz	0.32 cm*	8.1 cm	32 cm
f = 15 MHz	0.97 cm*	24 cm	97 cm

**unacceptably short near zone length*

(b) Diffraction limit (divergence angle θ):

	A = 1 mm	A = 10 mm	A = 20 mm
f = 2 MHz	28°*	5.4°*	2.7°
f = 5 MHz	11°*	2.2°	1.1°
f = 15 MHz	3.6°	0.71°	0.35°

**unacceptably large divergence angle*

11. (a) Calculate the beamwidth at focus for a 4 MHz transducer with an aperture of 2 cm focussed at a depth of 5 cm. (b) Repeat the calculation for the case where the same transducer is focussed at a depth of 15 cm.

(a) $(2.44 \times \lambda \times F)/A = 2.3$ mm

(b) 7.0 mm

12. Ultrasound is used to image a tissue interface at a depth of 8 cm. Most of the tissue through which the ultrasound passes has a propagation speed of 1540 m/sec, but there is a fat layer of 2 cm thickness adjacent to the face of the transducer with a propagation speed of 1450 m/sec. At what depth will the machine display the tissue interface?

Calculate the round-trip travel time of the ultrasound through the 6 cm of normal tissue and 2 cm of fat separately, then add them:

$t_1 = (2 \times 6 \text{ cm})/(1.54 \times 10^5) \text{ cm/sec} = 77.92\ \mu\text{sec}$

$t_2 = (2 \times 2 \text{ cm})/(1.45 \times 10^5) \text{ cm/sec} = 27.59\ \mu\text{sec}$

So the echo will arrive (77.92 + 27.59) = 105.51 µsec after the transmit pulse.

The displayed depth will be $(105.51\ \mu\text{sec} \times 1.54 \times 10^5 \text{ cm/sec})/2 = 8.12 \text{ cm}$

Thus the presence of the fat layer has caused the tissue interface to be displayed 1.2 mm deeper than its true position.

13. What is the lateral and axial resolution for a transducer with a beamwidth of 5 mm and a transmit pulse duration of 1.5 *µ*sec?

Lateral resolution = beamwidth = 5 mm

Axial resolution = $(1.54 \times 10^6 \text{ mm/sec} \times 1.5 \times 10^{-6} \text{ sec})/2 = 1.2 \text{ mm}$

Thus the lateral resolution is approximately 4 times worse than the axial resolution.

14. Calculate the Doppler shift for a 4 MHz Doppler system, a blood velocity of 80 cm/sec and a Doppler angle of 60°.

Doppler shift = 2.1 kHz

15. A 2.5 MHz pulsed Doppler is operating with a PRF of 4 kHz. (a) What is the Nyquist Limit? (b) If θ = 30°, what blood velocity would cause a Doppler shift equal to this Nyquist Limit?

(a) Nyquist Limit = 2 kHz

(b) Rearranging the Doppler equation gives

$v = (c \times f_D)/(2 \times f \times \cos\theta) = 71 \text{ cm/sec}$